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Exploring the Limits of Broadband Excitation and Inversion Pulses

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The design of broadband excitation and inversion pulses with compensation of B_1-field inhomogeneity is a long standing goal in high resolution NMR spectroscopy. Most optimization procedures used so far have been restricted to particular pulse families to keep the scale of the problem within manageable limits. This restriction is unnecessary using efficient numerical algorithms based on optimal control theory. A systematic study of rf-limited BEBOP (Broadband Excitation By Optimized Pulses) and BIBOP (Broadband Inversion By Optimized Pulses) pulses with respect to bandwidth and B_1-field is presented. Upper limits on minimum pulse lengths are set for different degrees of pulse performance.

Key Words: **optimal control theory; broadband excitation; broadband inversion; BEBOP; BIBOP.**

INTRODUCTION

The development of broadband excitation and inversion pulses has a long history. The hard pulse as the work horse of modern NMR spectroscopy is not compensated for B_1-field inhomogeneity and in the case of excitation causes a linear phase change with offset. While this phase variation can be compensated by a linear phase correction in one-dimensional experiments, more complex pulse sequences are expected to show significant improvements if pulses with uniform behavior over a range of offsets and rf-amplitudes could be used. Until now, the design of such pulses requires computer optimization, where the performance of a pulse can easily be simulated by rotations in three dimensional space. However, optimization algorithms applied so far for broadband excitation and inversion only lead to satisfying results for a relatively low number of independent optimization parameters, typically on the order of 10 to 100 parameters that define the pulse shape. As a consequence the space of possible pulse shapes in an optimization had to be reduced to certain pulse families characterized by a relatively small set of parameters, such as phase-alternating pulses [1, 2, 3, 4], symmetric pulses [5, 6], pulses with constant amplitudes [7, 8], Gaussian pulse cascades [9], Hermite polynomials [10], Fourier-series based pulses [11, 12, 13], spline interpolated pulse shapes [14], adiabatic pulses [15, 16, 17, 18] or combinations thereof [19, 20].

A different optimization procedure that was first used in magnetic resonance for the optimization of band-selective pulses in MRI [21, 22, 23] is based on optimal control theory. In this procedure a gradient towards better performing parameters is calculated efficiently based on an analytical formula that allows a significant increase in the number of independently optimized parameters. The adaptation of optimal control theory to the problem of broadband excitation pulses including the optimization with limited rf-amplitude was reported recently [24, 25]. Because of the high efficiency of the algorithm the space of possible pulse shapes used in the optimization is not restricted to any pulse family. Its fast convergence allowed the coverage of 8000 independently optimized parameters in reasonable time [24]. In spin systems where the theoretical limits of quantum evolution were known [26, 27, 28, 29, 30, 31, 32], numerical algorithms based on principles of optimal control theory, provide pulse sequences which approach the physical limits [33, 34, 35].

An optimal control theory based numerical algorithm therefore appears to be a well-suited tool to explore the physical limits for robust broadband excitation and inversion. In particular, we specify upper limits for the minimum durations of pulses as a functions of bandwidths and rf variation.

THEORY

Optimal control theory and its application to NMR spectroscopy is described in detail elsewhere [22, 23, 24, 25, 35]. The optimization algorithm used in the systematic studies of excitation and inversion pulses presented in this article is identical to the one used in [25] to limit the maximum rf-amplitude.

The quality factor or final cost $\mathbf{\Phi}$ used in the optimizations is the transfer efficiency from the initial magnetization $\mathbf{M}(t_0) = \mathbf{M}_z$ to the target state $\mathbf{F}$ ($\mathbf{F} = \mathbf{M}_x$ for excitation and $\mathbf{F} = -\mathbf{M}_z$ for inversion pulses) averaged over all offsets and rf-amplitudes specified for a specific optimization. It can be written as

$$\mathbf{\Phi} = \frac{1}{n_{\mathrm{off}}\ n_{\mathrm{rf}}} \sum_{i=1}^{n_{\mathrm{off}}} \sum_{j=1}^{n_{\mathrm{rf}}} \mathbf{M}_{ij}(t_p) \cdot \mathbf{F} \qquad (1)$$

with $i = 1..n_{\mathrm{off}}$ being the offsets and $j = 1..n_{\mathrm{rf}}$ the scaled rf-amplitudes calculated for each pulse of length t_p, e.g. to include the effects of rf-inhomogeneity or rf-amplitude misadjustments. In all cases, the nominal (unscaled) rf-amplitude was limited to 10 kHz using the method described in [25]. Sets of excitation and inversion pulses were calculated for bandwidths of 10, 20, 30, 40, and 60 kHz considering both ideal rf amplitude (scale factor of 1) and a variation of ± 20 percent in the factor used to scale the rf amplitudes. Also sets of pulses for a fixed bandwidth of 20 kHz with variations ϑ of ± 10, ± 20, ± 30, and ± 40 percent in rf scale factor were optimized to test robustness against B_1-field inhomogeneity. For each set, pulse lengths t_p were varied in ranges as listed in Table 1. Generally, pulse durations were incremented until the quality factor $\mathbf{\Phi}$ exceeded 0.995. Each chosen bandwidth was divided into equal increments, with $n_{\mathrm{off}} = 100$ for 10 kHz bandwidth, $n_{\mathrm{off}} = 200$ for bandwidths of 20, 30, and 40 kHz and $n_{\mathrm{off}} = 300$ for 60 kHz bandwidth. n_{rf} was chosen equal to 5 with equidistant percentage amplitude changes whenever variations in rf-amplitude were included in the calculations.

100 randomized starting pulses were generated to start 100 optimizations for each data point in Figs. 1 and 2. The time digitization for the optimized shapes was 0.5 μs in all cases. For short pulses the algorithm rapidly converges and in practically all runs the optimum quality factor is obtained. In general, the optimal quality factor depends on the initially chosen (random) pulse, but a significant percentage of the optimizations converge to similar optimal values even in cases with tight constraints (see Fig. 3).

The convergence of every single optimization was very fast ranging from seconds for the shortest pulses to tens of minutes for the longest ones with larger n_{off} and n_{rf} on a single AMD Athlon 1500+ processor Linux-based PC.

RESULTS

The results of the optimizations of excitation and inversion pulses are shown in Figs. 1 and 2, respectively: The performance of the optimized pulses described by the quality factor $\mathbf{\Phi}$ is given as a function of pulse length on a linear scale in Figs. 1 A, D and 2 A, D. A logarithmic scale is used in Figs. 1 B, E and 2 B, E to show the differences at longer pulse durations more clearly. As expected, higher demands in terms of bandwidth or tolerance to rf-amplitude variation lead to reduced quality factors that can, however, be compensated by increased pulse lengths. In all cases pulses with more than 99.5 percent excitation or inversion over the entire offset and rf-amplitude ranges could be found for unexpectedly short pulse durations of significantly less than 700 μs. A question of considerable practical interest is the minimum pulse length needed to achieve an excitation or inversion of a given quality. This information is shown in Figs. 1 C, F and 2 C, F. The relation between the duration and bandwidth is roughly linear for both types of pulses for the investigated offset and rf ranges.

The dependence of the quality factor on the pulse duration is not a smooth curve, but shows steps or shoulders (c.f. Figs. 1 A,D and 2 A,D). A more detailed analysis reveals that these steps are related to specific pulse families. Representatives of such pulse families are shown in Fig. 4 for excitation. For very short pulses, optimal control theory found that hard pulses, with constant amplitude and phase, provided the best performance. For longer pulses, the algorithm introduced 180° phase shifts resulting in pulses very similiar to phase-alternating composite pulses with one, two, and three phase jumps. If the pulse length increases further, the phase jumps 'morph' into more continuous phase changes. As was shown in [25] even longer pulses will have significant amplitude modulation with some periods of maximum rf-amplitude and might eventually not even reach the rf-limit [24]. While for the long pulses a distinction cannot be given, the pulse families shown in Fig. 4 correspond each to a 'step' in Fig. 1 A, D.

Pulses optimized for inversion all show constant rf-amplitude over the entire length, but still the observed

TABLE 1

Constraints used for BEBOP and BIBOP optimizations

rf-limit	$\Delta\nu^a$	n_{off}	ϑ^b	t_p excitation	t_p inversion
10 kHz	10 kHz	100	-	2.5 - 85 μs	2.5 - 122.5 μs
10 kHz	20 kHz	200	-	2.5 - 142.5 μs	2.5 - 182.5 μs
10 kHz	30 kHz	200	-	2.5 - 217.5 μs	2.5 - 247.5μs
10 kHz	40 kHz	200	-	2.5 - 267.5 μs	2.5 - 312.5 μs
10 kHz	60 kHz	300	-	2.5 - 422.5 μs	2.5 - 457.5 μs
10 kHz	20 kHz	200	± 10 %	2.5 - 247.5 μs	2.5 - 242.5 μs
10 kHz	20 kHz	200	± 20 %	2.5 - 337.5 μs	2.5 - 307.5 μs
10 kHz	20 kHz	200	± 30 %	2.5 - 442.5 μs	2.5 - 352.5 μs
10 kHz	20 kHz	200	± 40 %	2.5 - 667.5 μs	2.5 - 417.5 μs
10 kHz	5 kHz	100	± 20 %	30 - 60 μs	40 - 115 μs
10 kHz	10 kHz	200	± 20 %	50 - 125 μs	50 - 170 μs
10 kHz	20 kHz	200	± 20 %	100 - 190 μs	85 - 212.5 μs
10 kHz	30 kHz	300	± 20 %	150 - 285 μs	200 - 250 μs
10 kHz	40 kHz	300	± 20 %	180 - 405 μs	265 - 385 μs

[a] $\Delta\nu$ is defined as the excitation/inversion bandwidth used in the optimization. [b] ϑ is the range of rf amplitude scaling incorporated in the optimization.

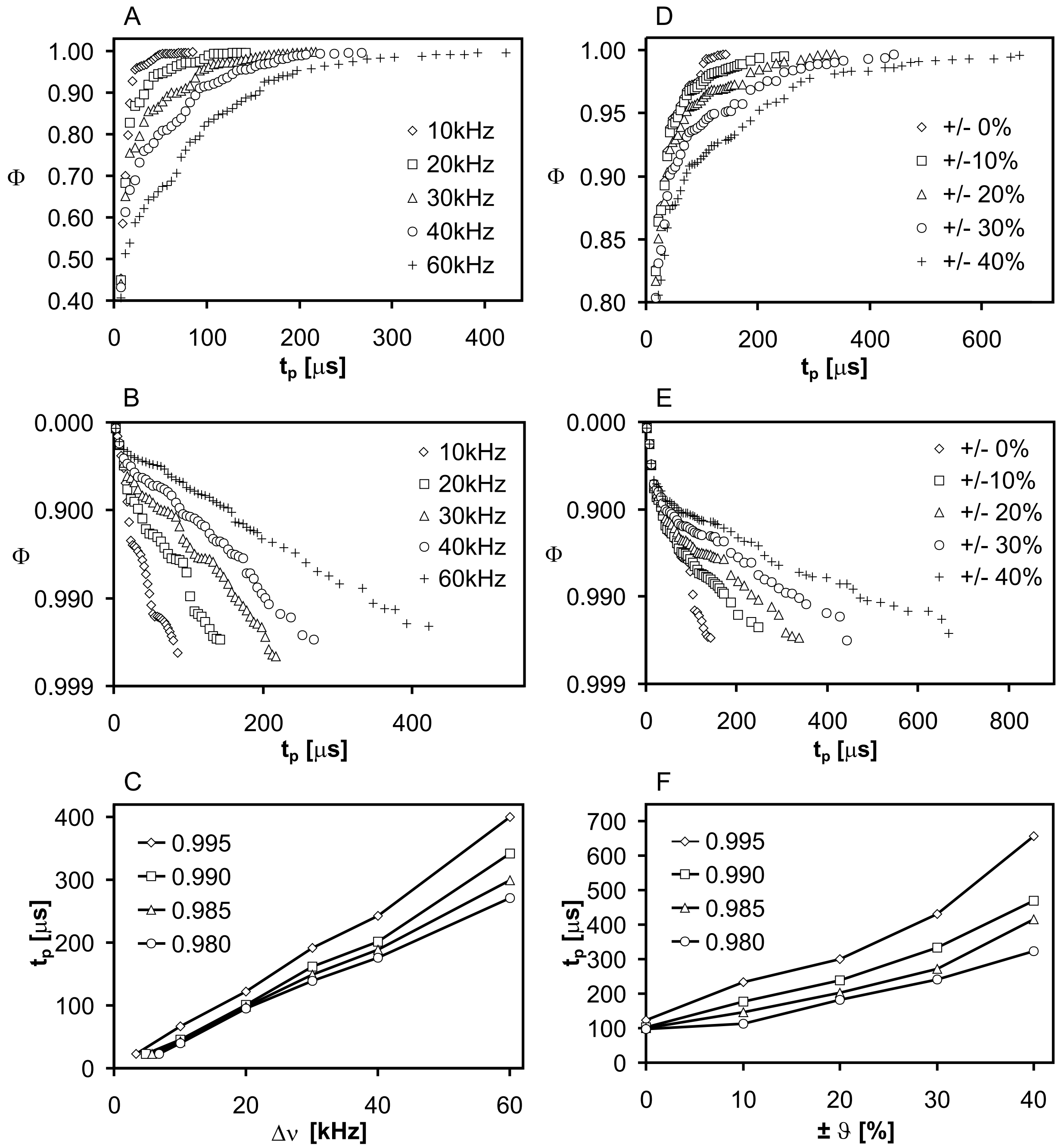

FIG. 1. Maximum quality factors reached for broadband excitation pulses (BEBOP) with rf-amplitude limited to 10 kHz under various optimization constraints. The maximum quality factors Φ with respect to pulse duration is given for the five different bandwidths Δν equal to 10 kHz, 20 kHz, 30 kHz, 40 kHz, and 60 kHz on a linear (A) and logarithmic scale (B). In C, the pulse lengths for quality factors of 0.98, 0.985, 0.99, and 0.995 are plotted as a function of the desired bandwidth and provide an estimate for the minimum pulse duration needed for specific requirements. The maximum quality factors Φ with respect to rf-variation are shown for no variation and rf-ranges ϑ of ±10 %, ±20 %, ±30 %, and ±40 % on a linear (D) and logarithmic scale (E) for a fixed bandwidth of 20 kHz. The dependence on minimum pulse duration for quality factors of 0.98, 0.985, 0.99, and 0.995 with respect to rf-variation is shown in (F).

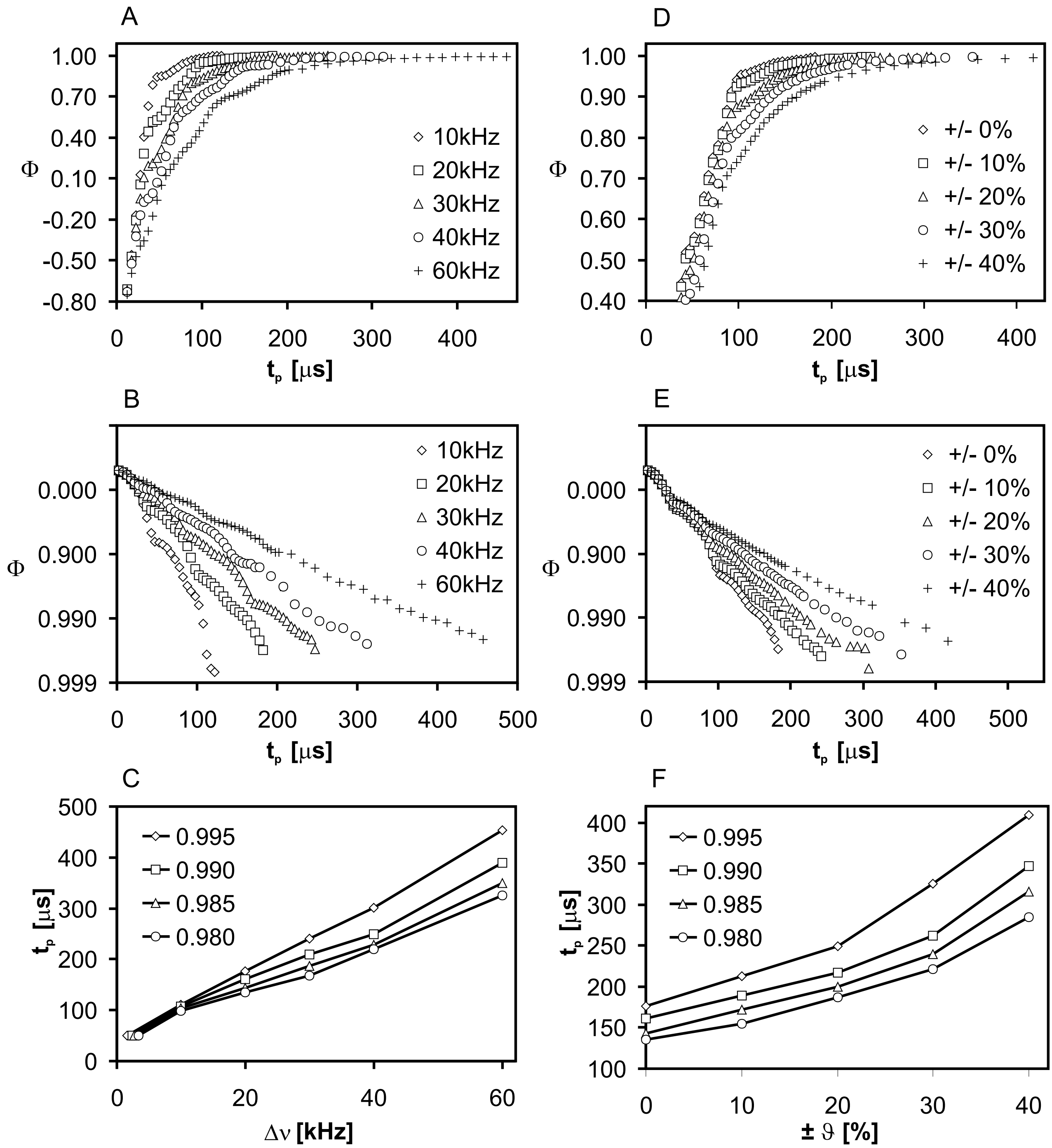

FIG. 2. Maximum quality factors reached for broadband inversion pulses (BIBOP) with rf-amplitude limited to 10 kHz under various optimization constraints. Again, maximum quality factors Φ with respect to pulse duration are given for the five different bandwidths $\Delta\nu$ equal to 10 kHz, 20 kHz, 30 kHz, 40 kHz, and 60 kHz on a linear (A) and logarithmic scale (B). In C, the pulse lengths for quality factors of 0.98, 0.985, 0.99, and 0.995 are plotted as a function of the desired bandwidth and provide an estimate for the minimum pulse duration needed for specific requirements. The maximum quality factors Φ with respect to rf-variation are shown for no variation and rf-ranges ϑ of ± 10 %, ± 20 %, ± 30 %, and ± 40 % on a linear (D) and logarithmic scale (E) for a fixed bandwidth of 20 kHz. The dependence on minimum pulse duration for quality factors of 0.98, 0.985, 0.99, and 0.995 with respect to rf-variation is shown in (F).

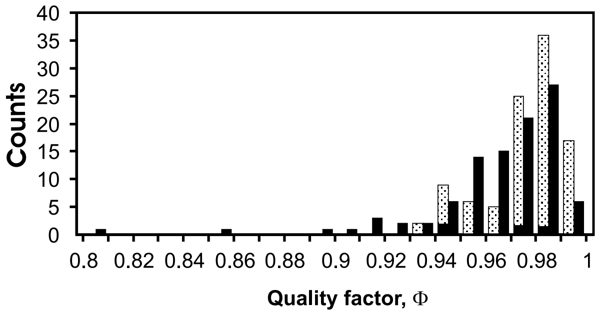

FIG. 3. Histogram distribution of quality factors Φ obtained for 100 optimizations for excitation pulses of 20 kHz bandwidth, of 300 μs duration, and ± 20 % rf-variation (dotted bars); 600 μs duration and ± 40 % variation in rf-amplitude (black bars). A significant percentage of the optimizations is close to the maximum quality factors of 0.9955 and 0.9957, respectively.

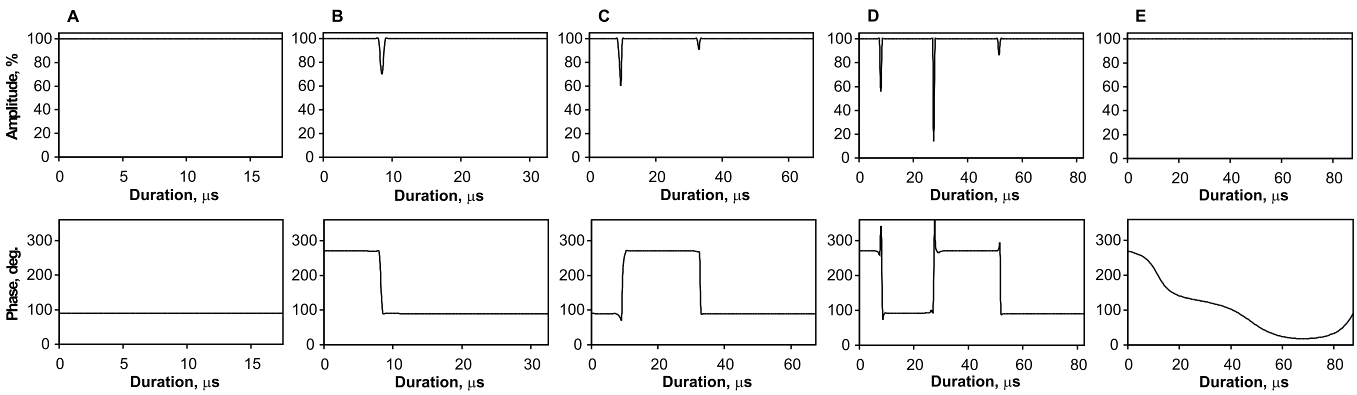

FIG. 4. Amplitude and Phase for optimized excitation pulses of various durations found in the optimization for a bandwidth of 30 kHz and no rf-variation. Although no restrictions to the pulse shape were made, phase-alternating composite pulses are found for pulse durations of up to 82.5 μs, while smooth phase-modulations were found for longer pulses.

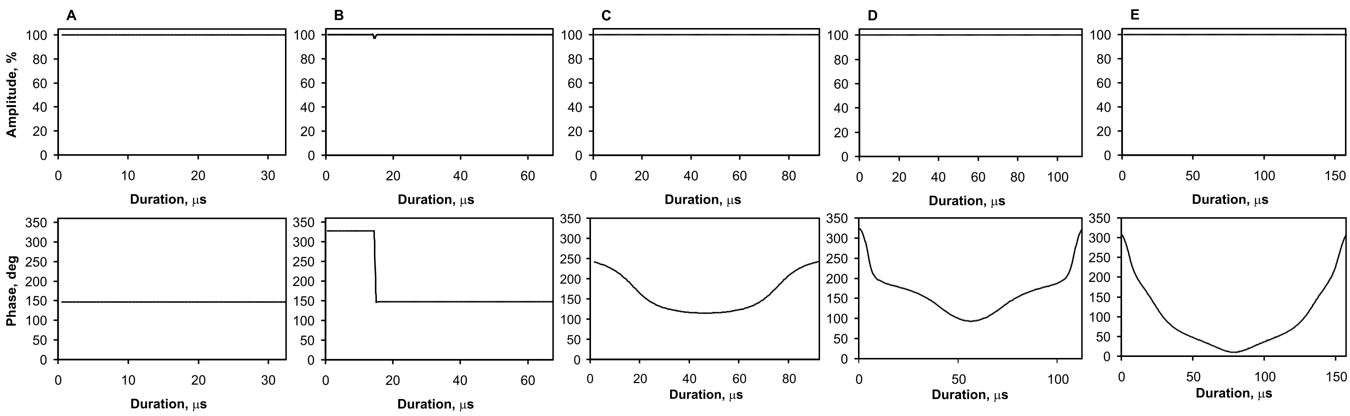

FIG. 5. Amplitude and phase behavior for optimal inversion pulses of various durations found for an optimization bandwidth of 20 kHz and no rf-variation. Constant amplitude pulses were obtained for all pulse lengths. While hard and phase-alternating composite pulses appear to be optimal for very short durations, a symmetric class of pulses with smooth phase modulations seems to be optimal for pulse lengths longer than 70 μs.

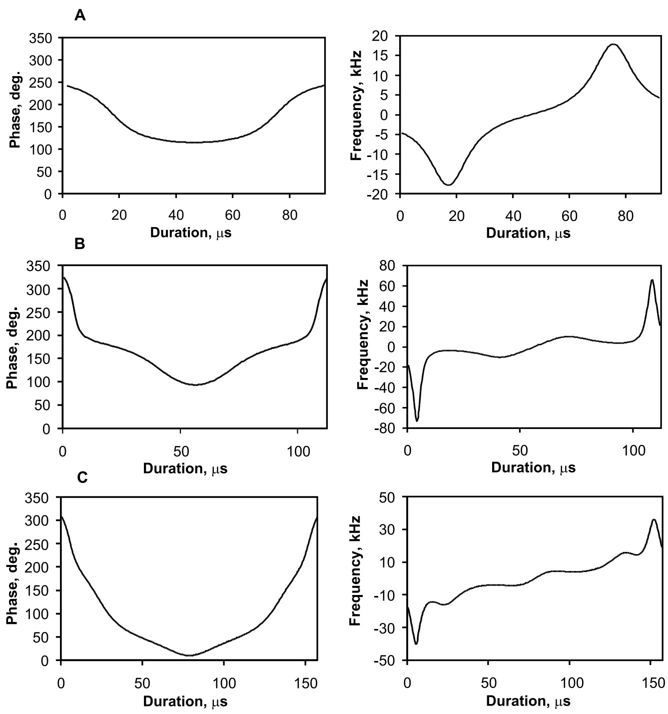

FIG. 6. The phase behavior of the pulses shown in Fig. 5 C-E has been converted to frequency modulation for comparison. A symmetric non-linear frequency sweep is observed. While the frequency sweep observed in the pulse center is reminiscent of adiabatic pulses, the swings at the pulse edges are not of high adiabaticity.

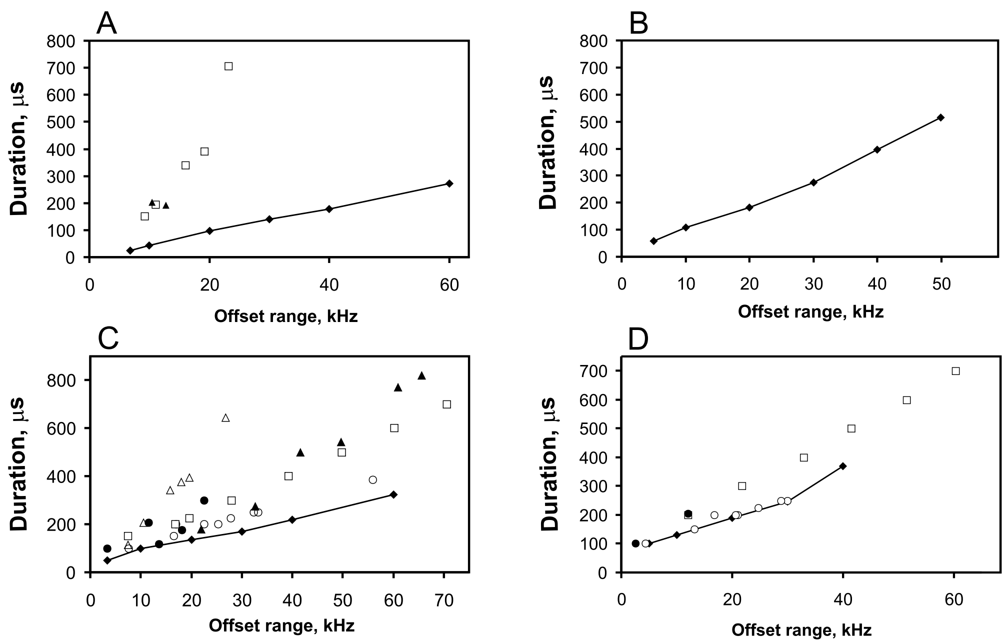

FIG. 7. Comparison of the maximum bandwidths with a quality factor **Φ** of 0.98 for previously reported broadband excitation (A,B) and inversion pulses (C,D) relative to BEBOP and BIBOP pulses obtained here. BEBOP and BIBOP pulses are indicated by filled diamonds which are connected by solid lines. (A) For excitation, BEBOP pulses were compared with pulses from [6] (squares) and other pulses cited in [36] from original references [1, 3] (filled triangles). (B) By taking a rf-variation of ±20 % into account, none of the composite pulses reached a quality factor of 0.98. (C) Inversion pulses compared to BIBOP were taken from [37] (filled triangles), [6] (open triangles), [20] (open circles), [38] (open squares), and other inversion pulses cited in [36] from original references [1, 4, 7, 8]. Only a $90_y240_x90_y$ pulse reaches the performance of BIBOP pulses. (D) The same comparison including ±20 % rf-variation. Only few composite pulses, adiabatic and BIP pulses reach a quality factor of 0.98. The shapes of the BIP pulses [20] are almost identical to the optimum BIBOP pulses.

step-like behavior corresponds to certain pulse patterns. Again hard pulses and pulses with a single 180° phase jump give best results for very short pulse durations, but a pulse family of pulses with smoothly modulated phase takes over for slightly longer pulses. The number of modulations in the phase marks different subclasses that again correspond to slight steps in Fig. 2 A, D. Although no symmetry constraints were imposed in the optimization, pulses of this class are perfectly symmetric around the pulse center. In Fig. 6 the phase and frequency of three such pulses is shown. The central smooth frequency sweep is strongly reminiscent of adiabatic pulses, which have a high degree of tolerance to rf inhomogeneity or miscalibration. However, the class of pulses derived here has constant maximum amplitude and therefore shows very low adiabaticity at the pulse edges. Instead, a pronounced frequency swing is observed that seems to achieve a similar effect as the amplitude modulation at the edges of adiabatic pulses. In general, the optimized pulses are similar to BIP pulses derived in [20] with only slightly improved inversion properties.

We also studied pulses only optimized for onresonant excitation (data not shown). The inclusion of B_1-field compensation in the optimization resulted in pulses with smooth phase modulations for pulses shorter than a 90° hard pulse. For slightly longer pulses even amplitude modulation can be observed.

For comparison of the BEBOP and BIBOP pulses with already known excitation and inversion pulses we calculated the quality factor $\mathbf{\Phi}$ for a large number of short published pulses. In all cases we set the maximum rf-amplitude to 10 kHz for consistent results. In addition, we set up comparisons without considering rf-amplitude variation and considering a variation ϑ of ±20 %. For all pulses we numerically determined the maximum bandwidth in which the quality factor $\mathbf{\Phi}$ reaches 0.98. In detail, composite pulses we used for comparison were taken from [1, 3, 6] for excitation and from [1, 4, 6, 7, 8, 37] for inversion. In addition we derived optimum sech/tanh and tanh/tan adiabatic pulses for several bandwidths as described in [38] and implemented most of the BIP inversion pulses [20]. The results are shown in Fig. 7: In the case of excitation no pulse reaches the performance of BEBOP pulses, considering variations in rf-amplitude none of the tested pulses does reach a quality factor of 0.98 (Fig. 7 A,B). In the case of broadband inversion, only the $90_y 240_x 90_y$ pulse [1] achieves the limit when no rf-variation is considered (Fig. 7 C). As shown in Fig. 7 D, the BIP pulses [20] closely approach the limits found by our algorithm if rf-amplitude variations of ±20 % are included in the calculations.

DISCUSSION

Modern high resolution NMR spectrometers with very high magnetic fields result in large offset ranges that have to be covered by modern pulse sequences. Especially ^{13}C and ^{19}F nuclei with their large offset ranges pose problems to conventional hard pulses. But also weak ^{15}N pulses of common triple resonance probeheads, for example, make it impossible to cover the whole nitrogen spectrum of uniformly labeled nucleic acids. The increased offset ranges could in principle be covered by stronger hard pulses, but high frequencies close to 1 GHz limit the technically available maximum rf-amplitude. An effective alternative to the hard pulse for covering the necessary bandwidth therefore is urgently needed.

The development of cryogenic probe heads allows NMR-measurements with significantly improved signal to noise ratios. However, the large temperature gradient in such a probehead leads to a coil design with significantly increased B_1-field inhomogeneity compared to conventional probeheads. As a consequence the sensitivity gain due to the cryogenic cooling is reduced with every uncompensated pulse. With robust pulses that are compensated for strong variations in rf-amplitude this loss in sensitivity could be strongly reduced.

Optimal control theory is an ideal tool for the design of pulses that fulfill the needs of modern NMR spectroscopy. The results shown in this article are the first systematic studies without restrictions to the pulse shape that allow the exploration of physical boundaries of the performance of excitation and inversion pulses. With these limits known it seems possible to optimize tailored pulses for specific needs in a reasonable time.

Some of the limitations to the applicability of the presented BEBOP and BIBOP pulses should be pointed out. All pulses are optimized starting with initial $\pm\mathbf{M}_z$ magnetization. The pulse is not defined for any other starting magnetization. However, if a BEBOP pulse shall be used to transfer $\mathbf{M}_x$ magnetization to $\mathbf{M}_z$, the time reversed pulse shape can be used. In addition, as with most other optimized excitation and inversion pulses, BEBOP and BIBOP pulses do not result in uniform unitary rotations. Initial magnetization components different from $\mathbf{M}_z$ will not be transfered the same way as a hard pulse would do. BIBOP pulses therefore cannot be used as refocussing pulses. The refocussing of a single transverse magnetization component (e.g. $-\mathbf{M}_x$) can be achieved by the application of two pulses, first a time-reversed and 180° phase shifted BEBOP pulse and then the original BEBOP pulse. In this case the magnetization component is refocussed in two steps: $-\mathbf{M}_x \rightarrow \mathbf{M}_z \rightarrow \mathbf{M}_x$. The development of pulses that act as uniform unitary rotations will be the subject of future research.

BEBOP and BIBOP pulses are, of course, scalable in the same way as conventional pulses. A pulse applied with twice the rf-amplitude will have half the duration and cover twice the bandwidth of the original pulse with the same robustness with respect to relative variations of the rf-amplitude. The data presented will therefore be useful as an estimate for most pulse requirements.

Besides practical aspects on the application of pulses in modern spectroscopy some theoretical aspects of the work presented should be noted. The length of a well B_1-field compensated excitation pulse exceeds the time of an equally well compensated inversion pulse. We therefore conclude that a controlled phase in the transverse plane in the presence of B_1-field inhomogeneity is a rather difficult task especially if the bandwidth exceeds the maximum allowed rf-amplitude. This complies with the results obtained for BEBOP pulses optimized for onresonant excitation only which show the pure effect of B_1-field compensation on excitation pulses (data not shown). Compensation for variation in rf-amplitude introduces phase modulation already for pulses of the length of a hard 90° pulse and amplitude modulation for slightly longer pulses. It seems that the phase-alternating composite pulses obtained from broadband studies therefore are a result of the bandwidth only, while B_1-compensation could be better achieved by smooth phase and amplitude changes.

Finally, for relatively short inversion pulses a symmetric class of pulses with constant amplitude appears to be optimal. The pulses show a frequency sweep similar to adiabatic pulses in the center but a distinct 'frequency swing' at the pulse edges and closely resemble the BIP pulses [20].

CONCLUSION

A very efficient algorithm based on optimal control theory was used to explore the physical limits of excitation and inversion pulses that does not pose any restrictions to the pulse form except for limiting the maximum rf-amplitude and the temporal digitization of the pulses. Steps in the performance of pulses with respect to pulse length could be related to certain types of pulses, including a class of relatively short inversion pulses with good compensation for variations in rf-amplitude. Correlations of the minimum pulse length required with respect to excitation/inversion bandwidth and compensation for B_1-field inhomogeneity are given. For almost all pulse lengths improvements relative to already published pulses could be achieved. The pulses will be made available for download on the website http://org.chemie.tu-muenchen.de/people/bulu .

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